Tema: Limites com funções exponenciais e logarítmicas

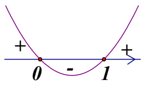
Proposta de Correcção

1. $\lim\limits_{x\to 1^+} \dfrac{e^x}{x-1} = \dfrac{e^1}{0^+} = +\infty$

2. $\lim\limits_{x\to 1^-} \dfrac{e^x}{x-1} = \dfrac{e}{0^-} = -\infty$. Logo $\lim\limits_{x\to 1} \dfrac{e^x}{x-1}$ não existe, porque os limites laterais são diferentes. (Ver limite calculado em 1.)

3. $\lim\limits_{x\to +\infty} \dfrac{e^x}{x-1} \overset{\left(\frac{+\infty}{+\infty}\right)}{=} \lim\limits_{x\to +\infty} \dfrac{\frac{e^x}{x}}{1-\frac{1}{x}} = \dfrac{\lim\limits_{x\to +\infty} \frac{e^x}{x}}{\lim\limits_{x\to +\infty}\left(1-\frac{1}{x}\right)} = \dfrac{+\infty^{(*)}}{1-\frac{1}{+\infty}} = \dfrac{+\infty}{1-0} = +\infty$

   (*) Pelo limite notável

4. $\lim\limits_{x\to 1^+} \dfrac{e^x}{x^2-x} = \dfrac{e}{0^+} = +\infty$ Nota: 

5. $\lim\limits_{x\to 0^+} (2x - x\ln x) = 0 - 0\times(-\infty) = (0\times\infty) Ind$

   $\lim\limits_{x\to 0^+} (2x - x\ln x) = \lim\limits_{x\to 0^+} 2x - \lim\limits_{x\to 0^+} x\ln x = 0 - \lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}} = -\lim\limits_{x\to 0^+} \dfrac{-\ln\left(\frac{1}{x}\right)}{\frac{1}{x}} =$

   $= \lim\limits_{y\to +\infty} \dfrac{\ln(y)}{y} = 0$, pelo limite notável

   Mudança de variável : $y = \dfrac{1}{x}$

   Se $x \to 0^+$ então $y \to +\infty$

6. $\lim\limits_{x\to +\infty} (2x - x\ln x) \overset{(\infty-\infty)}{=} \lim\limits_{x\to +\infty} x(2-\ln x) = +\infty \times (2-(+\infty)) = +\infty\times(-\infty) = -\infty$

7. $\lim\limits_{x\to -\infty} \dfrac{e^x}{x-1} = \dfrac{e^{-\infty}}{-\infty-1} = \dfrac{0}{-\infty} = 0$

8. $\lim\limits_{x\to 0^+} \dfrac{\ln x}{x} = \dfrac{\ln(0^+)}{0^+} = \dfrac{-\infty}{0^+} = -\infty$

9. $\lim\limits_{x\to +\infty} \dfrac{\ln x}{x} = 0$ (limite notável)

10. $\lim\limits_{x\to+\infty}\left(\ln\left(x+\dfrac{1}{x}\right)-\ln x\right)\overset{(+\infty-\infty)}{=}\lim\limits_{x\to+\infty}\ln\left(\dfrac{x+\dfrac{1}{x}}{x}\right)=\lim\limits_{x\to+\infty}\ln\left(1+\dfrac{1}{x^2}\right)=\ln\left(1+\dfrac{1}{+\infty}\right)=\ln(1+0)=\ln 1=0$

11. $\lim\limits_{x\to+\infty}\left(1+3x^2e^{-x}\right)=1+3\times(+\infty)\times e^{-\infty}=1+(+\infty)\times 0\ Ind$

$$\lim_{x\to+\infty}\left(1+3x^2e^{-x}\right)=\lim_{x\to+\infty}1+3\times\lim_{x\to+\infty}\frac{x^2}{e^x}=1+3\times\frac{\lim\limits_{x\to+\infty}1}{\lim\limits_{x\to+\infty}\dfrac{e^x}{x^2}}=1+3\times\frac{1}{+\infty}=1+0=1$$

12. $\lim\limits_{x\to-\infty}\left(1+3x^2e^{-x}\right)=1+3\times(+\infty)^2\times e^{+\infty}=1+\infty\times(+\infty)=+\infty$

13. $\lim\limits_{x\to+\infty}\dfrac{\ln(3x)}{x}\overset{\left(\frac{+\infty}{+\infty}\right)}{=}\lim\limits_{x\to+\infty}\dfrac{\ln 3}{x}+\lim\limits_{x\to+\infty}\dfrac{\ln x}{x}=0+0=0$, pelo limite notável

14. $\lim\limits_{x\to0^+}x^2e^{\frac{1}{x}}=0\times e^{\frac{1}{0^+}}=0\times e^{+\infty}=0\times(+\infty)\ Ind$

Mudança de variável:

$y=\dfrac{1}{x}$ Se $x\to0^+$ então $y\to+\infty$

$\lim\limits_{x\to0^+}x^2e^{\frac{1}{x}}=\lim\limits_{y\to+\infty}\left(\dfrac{1}{y}\right)^2e^y=\lim\limits_{y\to+\infty}\dfrac{e^y}{y^2}=+\infty$, pelo limite notável

15. $\lim\limits_{x\to0}\dfrac{e^x-e^{3x}}{2x}\overset{\left(\frac{0}{0}\right)}{=}\lim\limits_{x\to0}\dfrac{e^x\left(1-e^{2x}\right)}{2x}=$

$\lim\limits_{x\to0}e^x\times\lim\limits_{x\to0}-\dfrac{e^{2x}-1}{2x}=e^0\times\left(-\lim\limits_{y\to0}\dfrac{e^y-1}{y}\right)=1\times(-1)=-1$

Mudança de variável: $y=2x$ Se $x\to0$ então $y\to0$

16. $\lim\limits_{x\to-\infty}\dfrac{1}{xe^x}=\dfrac{1}{-\infty\times e^{-\infty}}=\dfrac{1}{-\infty\times0}\ Ind$

$\lim\limits_{x\to-\infty}\dfrac{1}{xe^x}=\lim\limits_{y\to+\infty}-\dfrac{e^y}{y}=-\lim\limits_{y\to+\infty}\dfrac{e^y}{y}=-(+\infty)=-\infty$, pelo limite notável

Mudança de variável:

$y=-x$ Se $x\to-\infty$ então $y\to+\infty$

17. $\lim\limits_{x\to0}\dfrac{e^{x+1}-e}{x}\overset{\left(\frac{0}{0}\right)}{=}\lim\limits_{x\to0}\dfrac{e\left(e^x-1\right)}{x}=\lim\limits_{x\to0}e\times\lim\limits_{x\to0}\dfrac{e^x-1}{x}=e\times1=e$, pelo limite notável

18. $\lim\limits_{x\to 0}\dfrac{1-e^x}{3x}\overset{\left(\frac{0}{0}\right)}{=}\lim\limits_{x\to 0}-\dfrac{1}{3}\times\dfrac{e^x-1}{x}=\lim\limits_{x\to 0}-\dfrac{1}{3}\times\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=-\dfrac{1}{3}\times 1=-\dfrac{1}{3}$, pelo limite notável.

19. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{e^x-1}\overset{\left(\frac{0}{0}\right)}{=}\lim\limits_{x\to 0}\dfrac{\frac{e^{2x}-1}{x}}{\frac{e^x-1}{x}}=\lim\limits_{x\to 0}2\times\dfrac{\frac{e^{2x}-1}{2x}}{\frac{e^x-1}{x}}=\lim\limits_{x\to 0}2\times\dfrac{\lim\limits_{x\to 0}\frac{e^{2x}-1}{x}}{\lim\limits_{x\to 0}\frac{e^x-1}{x}}=2\times\dfrac{1}{1}=2$, pelo limite notável.

20. $\lim\limits_{x\to 0^+}\dfrac{e^{2x}-1}{x^2}\overset{\left(\frac{0}{0}\right)}{=}\lim\limits_{x\to 0^+}2\times\dfrac{e^{2x}-1}{2x}\times\dfrac{1}{x}=\lim\limits_{x\to 0^+}2\times\lim\limits_{x\to 0^+}\dfrac{e^{2x}-1}{2x}\times\lim\limits_{x\to 0^+}\dfrac{1}{x}=$

$2\times 1\times\dfrac{1}{0^+}=2\times(+\infty)=+\infty$

21. $\lim\limits_{x\to 2}\dfrac{x^2-4}{e^{x-2}-1}\overset{\left(\frac{0}{0}\right)}{=}\lim\limits_{x\to 2}\dfrac{x-2}{e^{x-2}-1}\times(x+2)=\lim\limits_{x\to 2}\dfrac{x-2}{e^{x-2}-1}\times\lim\limits_{x\to 2}(x+2)=\lim\limits_{y\to 0}\dfrac{y}{e^y-1}\times$

$\times\lim\limits_{x\to 2}(x+2)=\lim\limits_{y\to 0}\dfrac{1}{\frac{e^y-1}{y}}\times\lim\limits_{x\to 2}(x+2)=\dfrac{\lim\limits_{y\to 0}1}{\lim\limits_{y\to 0}\frac{e^y-1}{y}}\times\lim\limits_{x\to 2}(x+2)=\dfrac{1}{1}\times(2+2)=4$

Mudança de variável: $y=x-2$ Se $x\to 2$ então $y\to 0$

22. $\lim\limits_{x\to+\infty}\dfrac{\ln x}{5x}\overset{\left(\frac{+\infty}{+\infty}\right)}{=}\lim\limits_{x\to+\infty}\dfrac{1}{5}\times\lim\limits_{x\to+\infty}\dfrac{\ln x}{x}=\dfrac{1}{5}\times 0=0$, pelo limite notável.

23. $\lim\limits_{x\to 0^+}\dfrac{\ln(2x+2)}{x}=\dfrac{\ln 2}{0^+}=+\infty$

24. $\lim\limits_{x\to+\infty}\dfrac{x^2+x}{\ln x}\overset{\left(\frac{+\infty}{+\infty}\right)}{=}\lim\limits_{x\to+\infty}\dfrac{\frac{x^2}{x}+\frac{x}{x}}{\frac{\ln x}{x}}=\dfrac{\lim\limits_{x\to+\infty}(x+1)}{\lim\limits_{x\to+\infty}\frac{\ln x}{x}}=\dfrac{+\infty}{0^+}=+\infty$